AENG 411: Aerospace Laboratory

Determination of the Natural Frequency of a Torsional Pendulum and Comparison to Thoery

by

Group No. 2

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# **Introduction**

A torsional pendulum consists of a long, stiff shaft fixed that is fixed at one end on attached to a large disk on the other. When said disk is rotated by a small angle and then released, the shaft will begin to rotate in a periodic fashion, the amplitude and frequency of which are highly reliant upon the moment of inertia and weight of the disk, as described in Equations (1-2), where *D*, *t*, *G*, *l*, and *ρ* are the respective diameter, thickness, shaft shear modulus, length, and density of the disc.

(1,2)

Based on these parameters, the equation of motion associated with the rotation of the shaft can be described by Equation (3), where *kr* is the rotational spring constant of the shaft, as described in Equation (4).

(3,4)

Based on these equations, the natural frequency and normal frequency can be described by Equations (5-6). Each of these correspond to the undamped frequency of oscillation of the shaft, so if one wants to find the damped frequency of oscillation, Equation (7) can be used, where the damping ratio, *ς*, is defined by Equation (8), where the logarithmic decrement, *δ*, is defined by Equation (9). The u values listed in Equation (9) correspond to amplitudes of oscillation at two separate points (i and i+j). Experimentally, these points can be considered at amplitudes of 90⁰ and 45⁰ respectively.

(5-9)

With all of these values in mind, the behavior of the shaft, disk system when disturbed from equilibrium can be properly predicted and assessed.

# Design of Test

#### 2.1. Objective

The goal of this experiment is to compare the measured natural frequency to the theoretical natural frequency.

#### 2.2. Test Apparatus and Function

The measurement tools and apparatus that were necessary to run this experiment are listed below:

* Steel and Aluminum Torsional Pendulums
* Test Stand
* Strain Gauge
* Strain Indicator
* DAQ

# Computer with LabView Procedure

1. A quarter bridge connection was made between the strain gauge and the strain indicator.
2. The indicator was zeroed and the gauge factor was set to 2.
3. One of the pendulums was placed in the clamp and the clamp was tightened.
4. The dimensions of the pendulum were measured.
5. The disc was rotated and released.
6. The natural frequency, damping ratio, and damped frequency were recorded.
7. The disc was rotated for a total of six trials.
8. Repeated steps 4-7 for a different shaft length (Note: the second length for the steel rod was not run, as the strain indicator for it broke between tests)
9. Repeated steps 4-8 for the other material.

# Results

For this experiment, the length and diameter of each rod, as well as the thickness and diameter of the plate attached to said rod, used during testing were recorded, as shown in Table 4-1.

**Table 4-1. Rod/Disk Dimensions for Each Trial Run**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Trial Type/Number** | **Rod Length (in)** | **Rod Diameter (in)** | **Plate Thickness (in)** | **Plate Diameter (in)** |
| Steel 1 | 43.75 | 0.13 | 0.50 | 5.80 |
| Aluminum 1 | 38.81 | 0.13 | 0.50 | 6.00 |
| Aluminum 2 | 43.88 |

For all 4 trial runs associated with each length configuration, the natural frequency, damping frequency, and damping ratio data for each case was recorded, as shown in Table 4-2. This data will later be compared to the theoretical quantities associated with each particular configuration, as discussed in the next section.

**Table 4-2. Frequency and Damping Ratio Data for Each Configuration**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Trial Type/Number** | | **Natural Frequency (Hz)** | **Damping Frequency (Hz)** | **Damping Ratio** |
| Steel 1 | 1 | 11.781 | 17.781 | 0.00213941 |
| 2 | 11.786 | 11.786 | 0.001977642 |
| 3 | 11.7812 | 11.7811 | 0.002216811 |
| 4 | 11.7813 | 11.7813 | 0.002055716 |
| Aluminum 1 | 1 | 11.9473 | 11.9473 | 0.00063299 |
| 2 | 11.9557 | 11.9557 | 0.000164483 |
| 3 | 11.9572 | 11.9572 | 0.000307668 |
| 4 | 11.9543 | 11.9543 | 0.000620185 |
| Aluminum 2 | 1 | 11.2562 | 11.2562 | 0.00054268 |
| 2 | 11.2657 | 11.2657 | 0.000238577 |
| 3 | 11.2633 | 11.2633 | 0.000449644 |
| 4 | 11.2617 | 11.2617 | 0.000397172 |

# Discussion of Results

Only four trials instead of the planned eight for the first steel pendulum length were obtained and the data that was recorded was suspect. This is attributed to equipment failure, specifically the strain gauge on the pendulum shaft. Because the aluminum pendulum had a separate strain gauge, the data for the aluminum trials is not suspect.

The 90% confident mean population values of the frequencies and damping ratios listed in Table 4-3 were found using the “Student T-Distribution Method” described at the end of the Aerolab Structures Lab Manual. For each length and material, the number of sample points (v) was four. α was found using the equation:

(10)

From these values, Table A.4 in the Aerolab Structures Lab Manual was used to find the value of tα/2 = 1.533.

After this, the below equation was used to calculate the range of 90% confident mean population values for the frequencies.

(11)

The natural frequencies for the theoretical and experimental Aluminum lengths appear to be very similar, though the values for steel appear to be significantly different.

In order to find the logarithmic decrement, it was necessary to find *umi* , *um(i+1)* , and *j*. The lab manual recommended using the starting and ending amplitudes for *umi* , *um(i+1)*  and the number of cycles between 90° > φ > 45°. However, this range of times was not recorded during the experiment. Instead, the starting amplitude, ending amplitude, and total number of cycles for the 60 second observation period were used. The logarithmic decrement was calculated using Equation (9), while the damping ratio was found with Equation (8). The results of these calculations are shown in Table 5-1, which shows that the theoretical damping ratios were all greater than those listed in Table 4-2. This suggests that the bars used during experimentation have been worn down and become less stiff than their theoretical counterparts, leading to increased times associated with the reduction of the amplitudes of the bars oscillatory rotation.

**Table 5-1. Theoretical and Experimental Damped Frequencies**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Trial** | **Kr (in\*lbf)** | **Natural Frequency (Hz)** | **Frequency (Hz)** | **umi** | **um(i+1)** | **j** | **δ** | **ζ** |
| Steel 1 | 7.24 | 22.98 | 3.66 | 0.44 | 0.11 | 32 | 0.0433 | 0.0069 |
|  |
| Aluminum 1 | 2.42 | 12.15 | 1.93 | 0.4 | 0.32 | 35 | 0.0064 | 0.0010 |
|  |
| Aluminum 2 | 2.14 | 11.43 | 1.82 | 0.5 | 0.38 | 32 | 0.0086 | 0.0014 |
|  |

The damped natural frequencies found from the previous analysis were then compared to the mean values found from the statistical analysis of the experimental data. The results of this process can be found in Table 5-2. Similar to the results of the natural frequency calculations, the damped frequencies for both aluminum lengths appear to match closely to the experimental values, with a percent difference of 0.795% for the first length and 0.712% for the second. However, the experimental and theoretical values for the steel data contrast sharply, with a percent difference greater than 19%, which may have resulted from problems that arose in the functionality of the strain gauge used to collect the experimental data, as it was put out of commission before the second test of the steel rod could be run.

**Table 5-2. Theoretical and Experimental Damped Frequencies**

|  |  |  |  |
| --- | --- | --- | --- |
| **Trial** | **Theoretical Damped Frequency (Hz)** | **Experimental Damped Frequency (Hz)** | **Percent Difference (%)** |
| Steel 1 | 22.98 | 15.5812 | 19.180 |
| Aluminum 1 | 12.15 | 11.9570 | 0.795 |
| Aluminum 2 | 11.43 | 11.2648 | 0.712 |

Finally, an example of the oscillatory data collected throughout the running of the experiment is shown in Figure 5-1.

**Figure 5-1. Example of Torsional Pendulum Data for Trial One of Length One of the Aluminum Rod**

# Conclusion

This experiment made use of strain gauge indicators and the Labview Program to obtain data for a torsional pendulum. Steel and aluminum pendulums were used. Different diameters and lengths of the shafts were used as well as different thicknesses and diameters of the disc which was attached at the lower end of the shaft. The disc was rotated about the axis of the shaft through a small angle and then released. The experiment frequency was the damped frequency. The Labview Program also recorded the natural frequency and the damping ratio. Hand calculations of these were also done, as discussed in Section (5).

This experiment could be used to estimate the shear modulus of a shaft of unknown material. This can be done by first calculating the moment of inertia of the disk connected to the test rod, where r is the radius of the disk and m is its measured mass. This moment of inertia could then be used to find the shear modulus of the shaft through the use of Equation (12), where D is the diameter of the shat and W is the weight of the disk.

(12,13)

The shear modulus of the shaft could be found by a plotting the period of its oscillation squared vs it length. This plot would consist of a straight line whose slope would be the shear modulus of the shaft.

It is also possible to obtain the moment of inertia, I, of the disc about the axis of the shaft of a torsional pendulum without knowing either the composition or the dimensions of the disc. This can be done by a combination of experimental observation and mathematical manipulation of equations involving a torsional pendulum.

First, the damped frequency of the pendulum’s motion would be obtained from experimental data. This, along with kr (Found from Equation (4)) and the damped ratio of motion can be used to find the undamped natural frequency of the pendulum from Equation (7). Finally, the moment of inertia of the disc could be found through the use of Equation (5).

Regarding sources of error: As noted in the Section (5), the data for the steel pendulum was suspect due to failure of the strain gauge on the steel pendulum shaft. Because the aluminum pendulum had a separate strain gauge, the data for the aluminum trial was not suspect. Other sources of error could have come from differences in the angle of twist, ϴ, used throughout the experiment. The motion of the pendulum was also not soley due to rotation about its primary axis, as some classical pendulum motion occurred due to the necessity of intiating the motion of the pendulum, which could have caused energy to be lost to changes in its potential energy as opposed to just its rotational energy. In addition, there may have been some loss of elasticity of the shaft from wear and tear.

The object of the experiment was to study the behavior of a torsional pendulum and in that, the object was met. There was difficulty with the data from the steel pendulum due to failure of the strain gage on the steel shaft. One did learn that it is possible to obtain the modulus of elasticity of an unknown shaft material from use of a torsional pendulum. The moment of inertia may also be obtained for a disc of unknown dimensions and characteristics through the use of a torsional pendulum.